COORBITAL MOTION IN THE CO-PLANAR RTBP:

Family of Quasi-satellite periodic orbits

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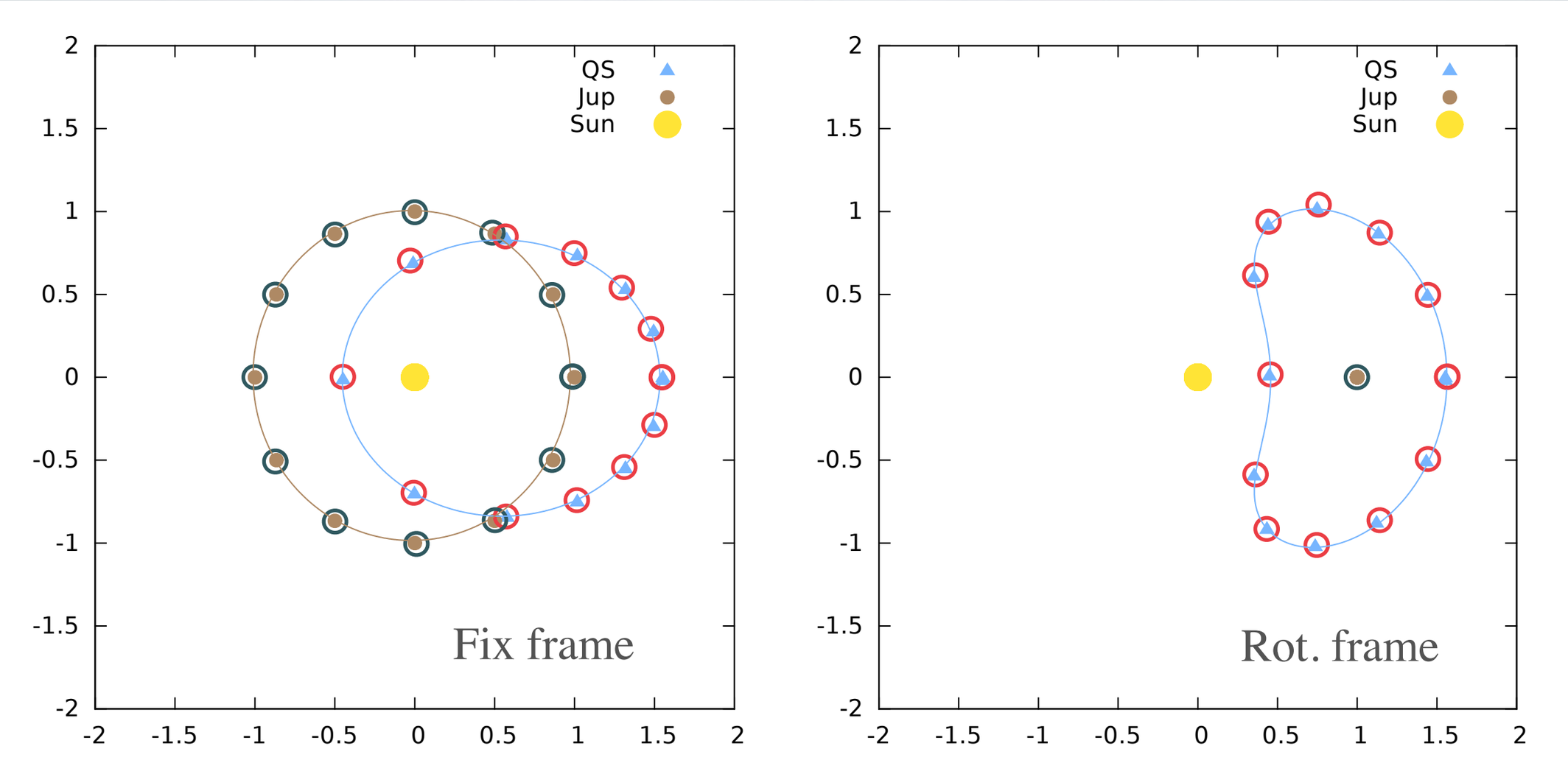
QS motion



Quasi-satellite definition:

- retrograde satellite outside the Hill sphere
- in heliocentric orbital elements:

co-orbital motion with the secondary

libration around $\theta = \lambda - \lambda_{pl} = 0$

Litterature

(non exhaustive)

Retrograde satellites:
Jackson 1913, Henon 1968, Benest 1977

QS (coorbital motion):
Mikkola & Innanen 1997, Namouni 1999, Sidorenko & al 2013

QS (coorbital objects in the SS):
Christou 2000, Brasser & al 2004, Kinoshita & Nakai 2007

QS (theorical studies on coorbital planets):
Hadjidemetriou & al 2004, Giuppone & al 2010

Outline

I. Context: Co-planar RTBP + averaging

II. The circular case

III. The family of QS periodic orbit in the circular case + co-orbital motion results

Context:

RTBP:

primary whose mass is equal to one
a secondary in circular or elliptic motion with a mass ϵ
a massless third body

+ co-planar motion
+ co-orbital resonance configuration

λ : mean longitude
a : semi major axis
e : eccentricity
ω : argument of periapsis
λ_{pl}: mean longitude of the planet
a_{pl}: semi major axis of the planet
e_{pl}: eccentricity of the planet
ω_{pl}: argument of periapsis of the planet
n_{pl}: mean motion of the planet

Hamiltonian Formalism:

$$\mathcal{H}(\lambda, a, e, \Delta\omega, \lambda_{pl}, \epsilon, a_{pl}, e_{pl}, n_{pl})$$

2 degrees of freedom
Non autonomous

parameters

$$\Delta\omega = \omega - \omega_{pl}$$

$$\lambda_{pl} = n_{pl} t$$

$$\theta = \lambda - \lambda_{pl} \qquad u = \frac{\sqrt{a} - \sqrt{a_{pl}}}{\sqrt{a_{pl}}}$$

Resonant variables

$$d\theta \wedge \sqrt{a_{pl}} u + \sqrt{a_{pl}}(1+u)(1-\sqrt{1-e^2}) \wedge d\Delta\omega$$

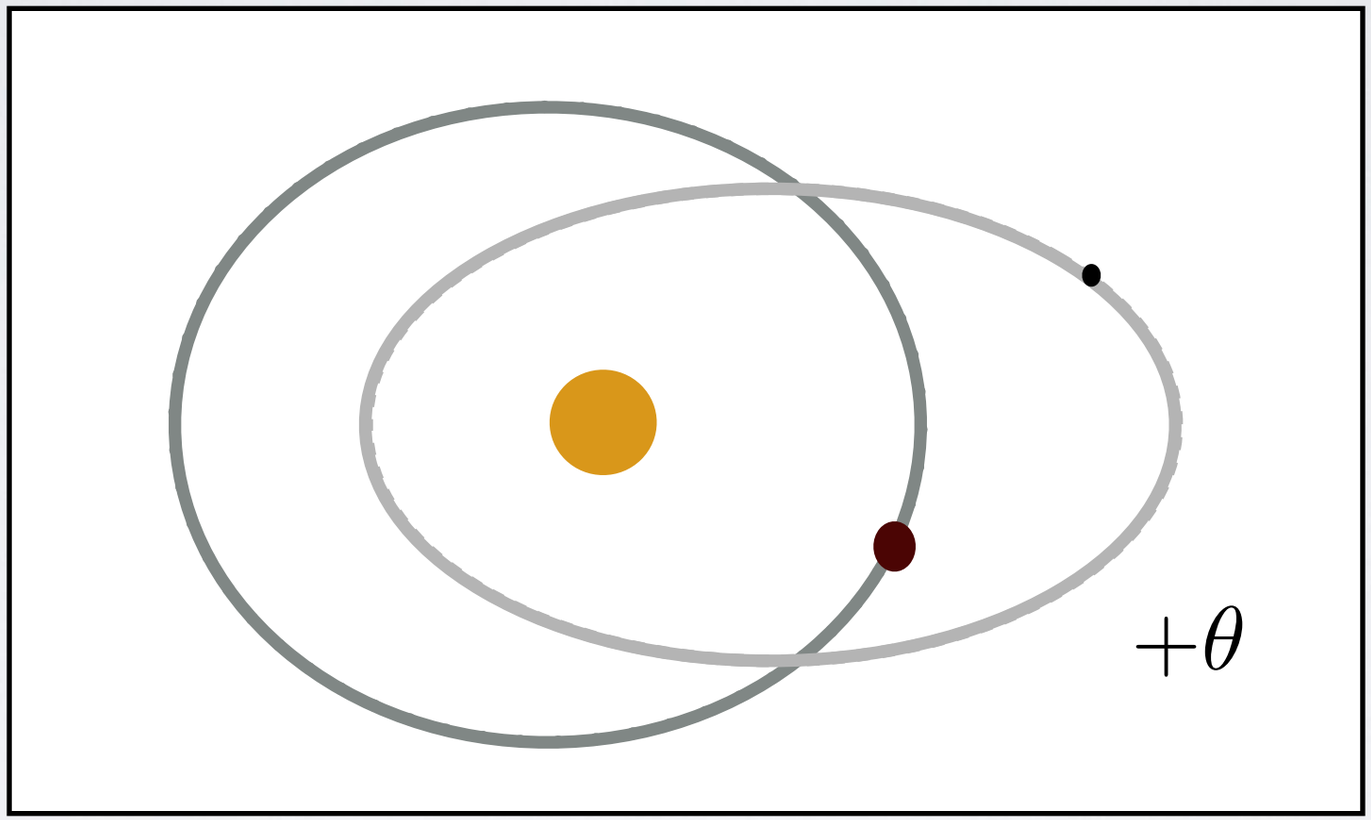
Symplectic form

+ adimensional problem $\quad a_{pl} = 1 \quad n_{pl} = 1$

$$\mathcal{H}(\theta, u, e, \Delta\omega, \lambda_{pl}; \epsilon, e_{pl})$$

Problem reduction:

averaging



Problem reduction:

averaging

Poisson Bracket

Symplectic transformation

$$\overline{\mathcal{H}} = \mathcal{H} + \epsilon\{\mathcal{W}, \mathcal{H}\} + o(\epsilon^2)$$

$$\overline{X} = X - \epsilon\{\mathcal{W}, X\} + o(\epsilon^2)$$

Average variable

Symplectic variable

Transformation generating function

$$\mathcal{H}(\theta, u, \Delta\omega, e; \epsilon, e_{pl}) = \mathcal{H}_0(u) + \epsilon\mathcal{H}_1(\theta, u, \Delta\omega, e, \lambda_p; e_{pl})$$

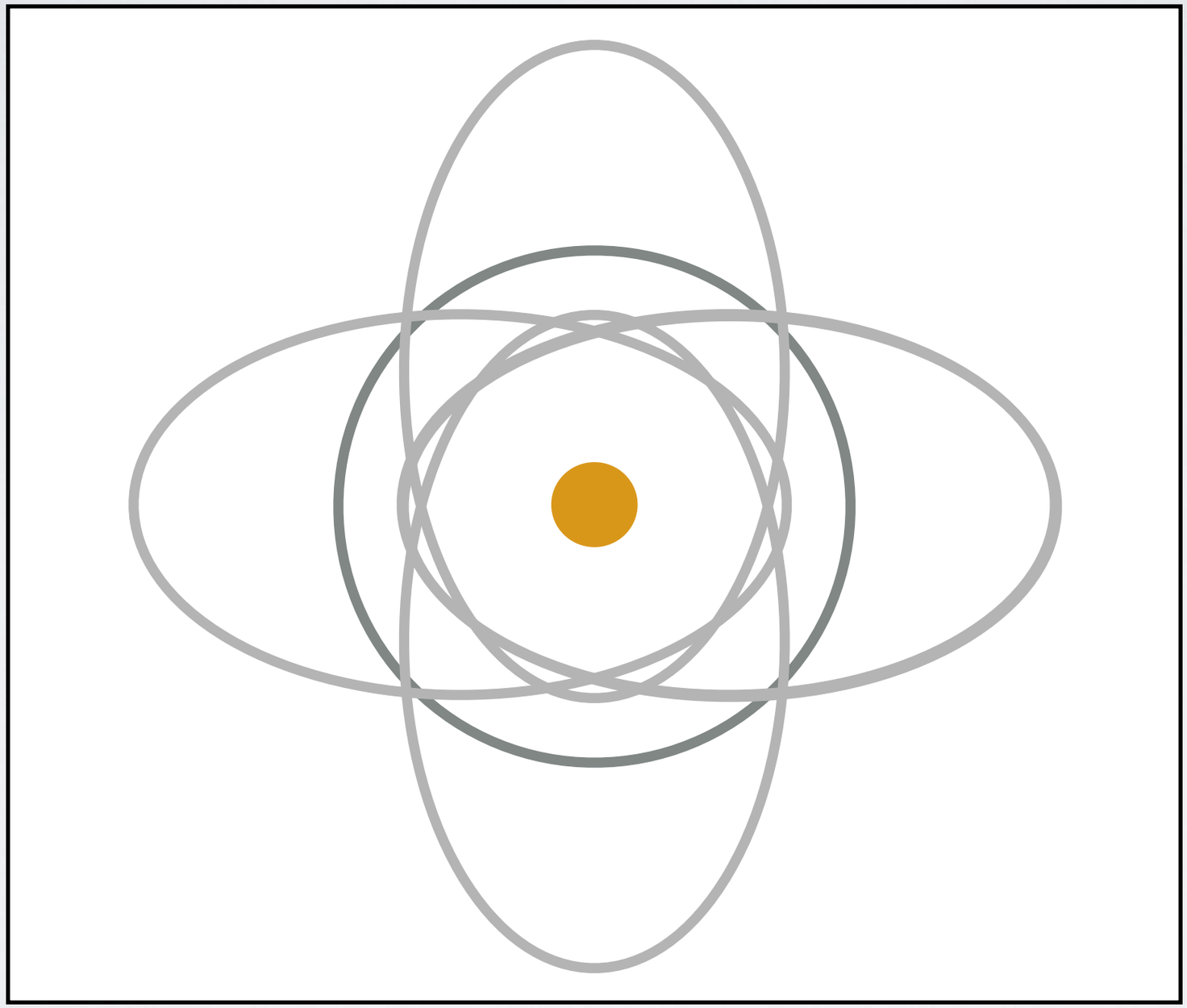
$$\overline{\mathcal{H}}(\theta, u, \Delta\omega, e; \epsilon, e_{pl}) = \underbrace{\mathcal{H}_0(u)}_{\overline{\mathcal{H}}_0} + \epsilon\underbrace{\frac{1}{2\pi}\int_0^{2\pi} \mathcal{H}_1(\theta, u, \Delta\omega, e, \lambda_{pl}; e_{pl})d\lambda_{pl}}_{\overline{\mathcal{H}}_1} + o(\epsilon^2)$$

2 degrees of freedom autonomous

$e_{pl} = 0$

Circular case

Invariance symmetry: Rotation



$e_{pl} = 0$

Circular case

Invariance symmetry: Rotation

$$\overline{\mathcal{H}}(\theta, u, \cancel{\Delta\omega}, e; \epsilon) \longrightarrow \begin{array}{l} \dot{\Gamma} = 0 \\ \Gamma = (1+u)(1-\sqrt{1-e^2}) \end{array}$$

2 degrees of freedom + one first integral

For a fixed Γ,
projection in (θ, u) plane contains all the co-orbital dynamic

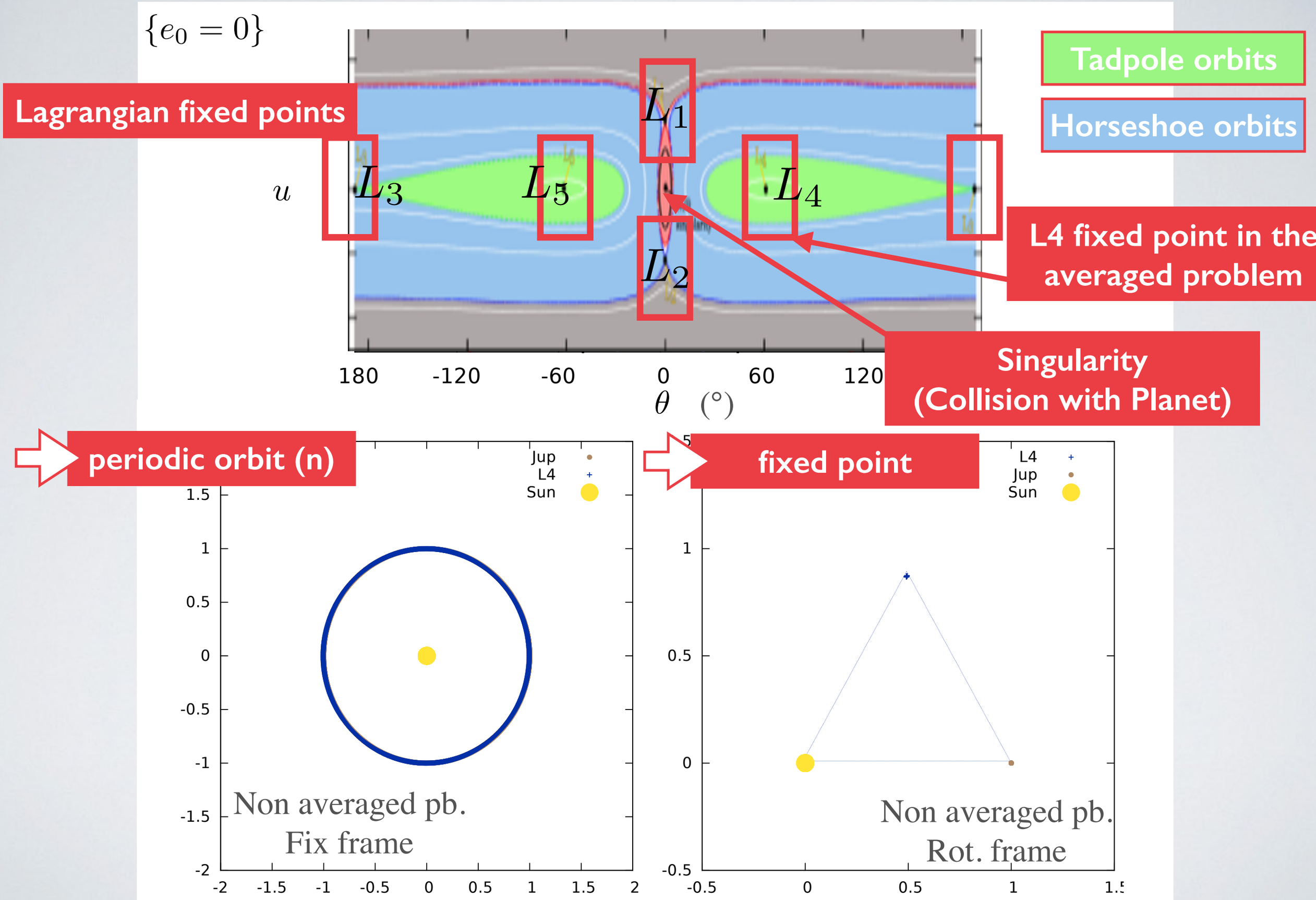
$$e = \sqrt{1 - \left(1 - \frac{\Gamma}{1+u}\right)^2} \rightarrow e_0 = \sqrt{1 - (1-\Gamma)^2}$$

seen as a parameter

$$\mathcal{H}(\theta, u, \boxed{e_0}; \epsilon)$$

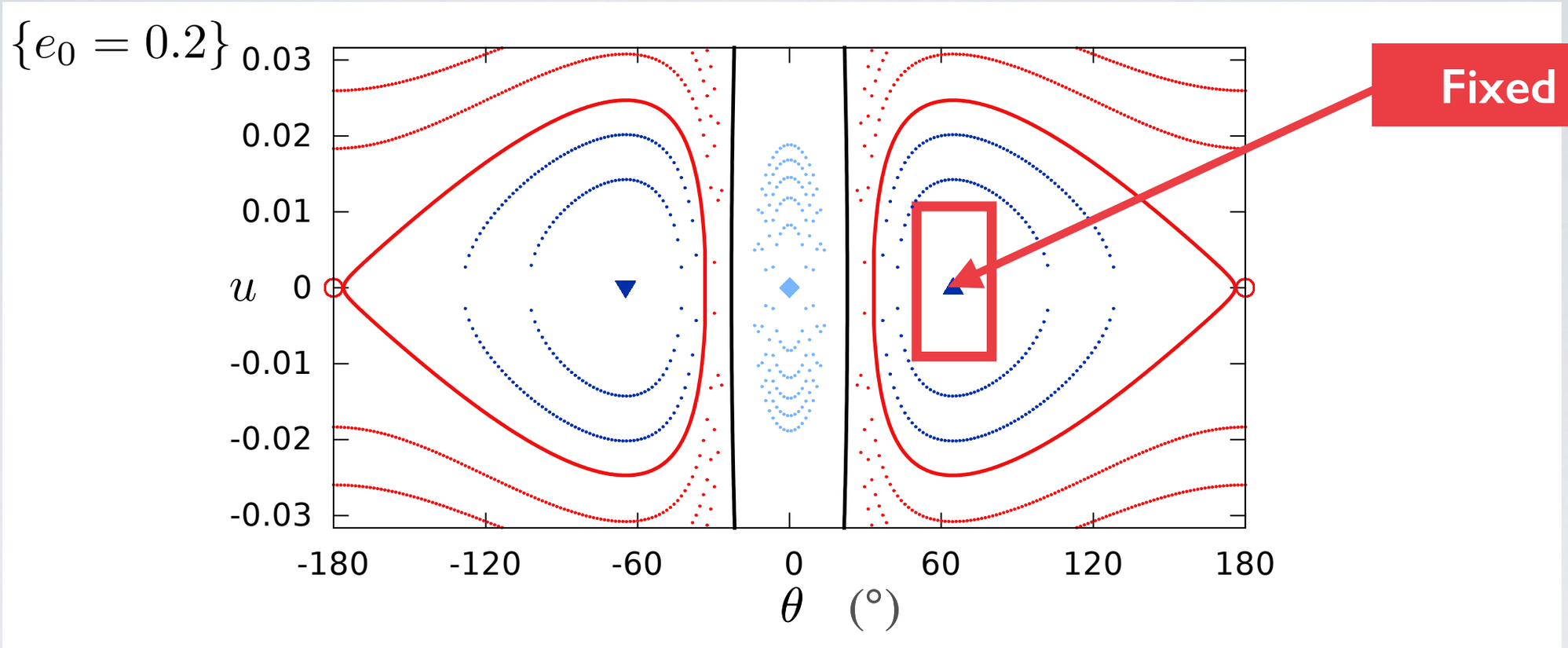
$e_{pl} = 0,\ \epsilon = 1/1000$

Averaged problem: (θ, u) projection



$e_{pl} = 0,\ \epsilon = 1/1000$

Averaged problem: (θ, u) projection



For $e_0 > 0$

an elliptical fixed point in the (θ, u)-plane has two frequencies:

g, precession of the argument of the periaster frequency

ν, libration in (θ, u)-plane frequency

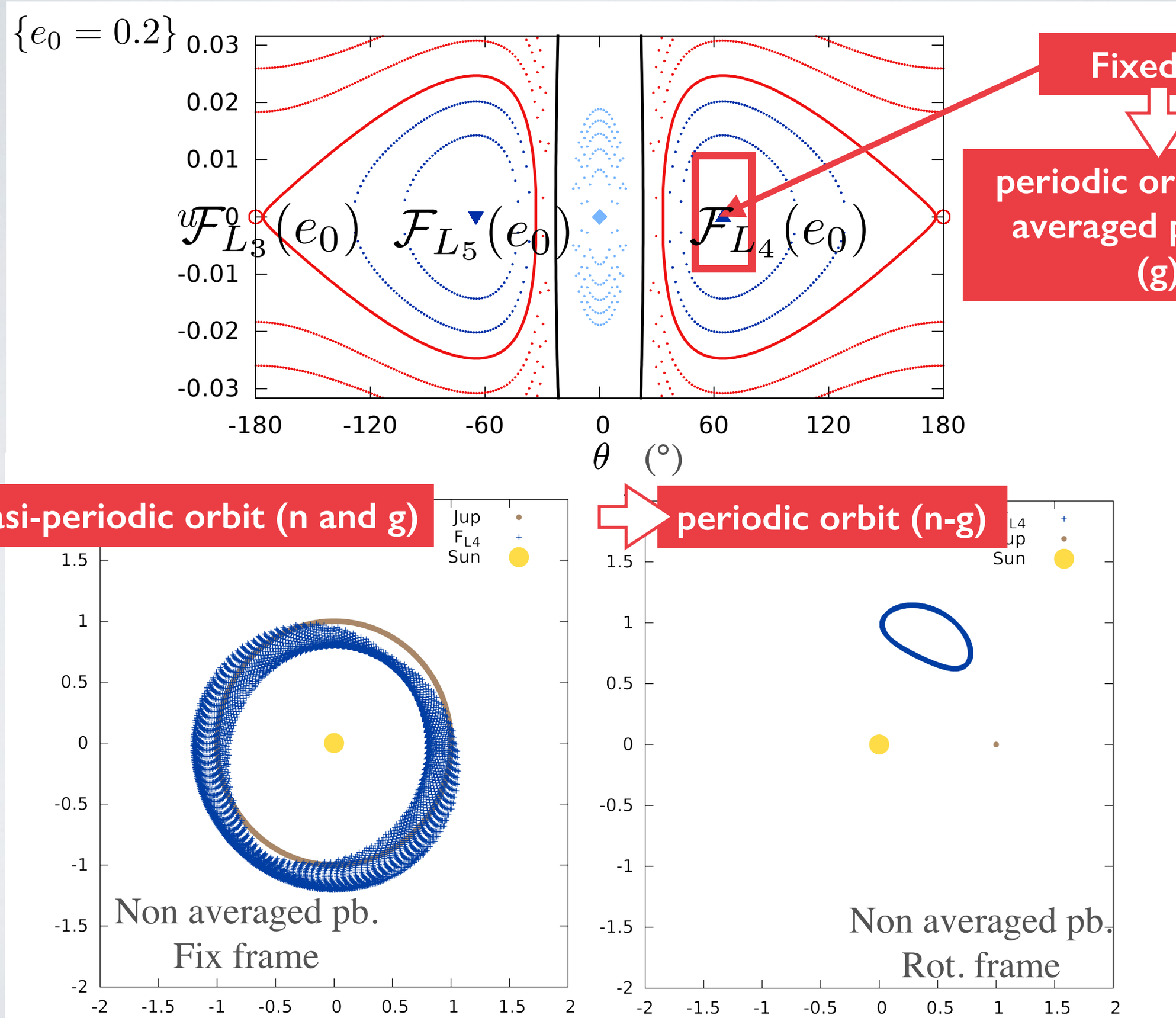
(around the fixed point)

An elliptic fixed point in the (θ, u)-plane is:

- **a periodic orbit in the averaged problem (g)**
- **a quasi-periodic orbit in the non-averaged problem [fix frame] (n and g)**
- **a periodic orbit in the non-averaged problem [rot. frame] (n - g)**

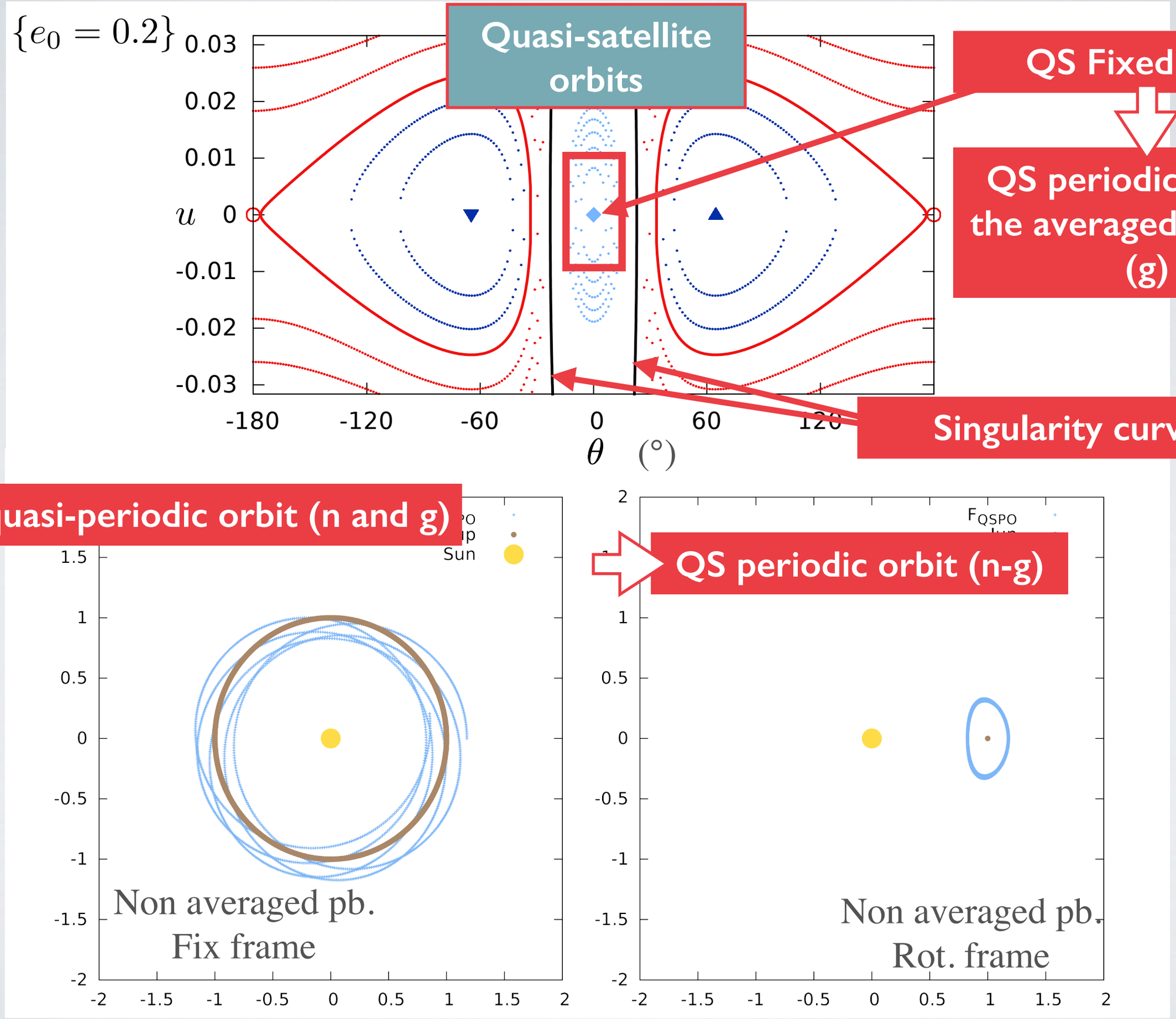
$e_{pl} = 0,\ \epsilon = 1/1000$

Averaged problem: (θ, u) projection



$e_{pl} = 0,\ \epsilon = 1/1000$

Averaged problem: (θ, u) projection



$e_{pl} = 0,\ \epsilon = 1/1000$ QS Periodic orbits study:

Method

Method developed by Nesvorny et al. (2002) to compute the averaged Hamiltonian and equations of motions.

+

Numerical Method to find a fixed point in (θ, u) plane (Newton-Raphson)

$$\dot{\theta}(\theta^*, u^*; e_0) = 0$$
$$\dot{u}(\theta^*, u^*; e_0) = 0$$

$$\boxed{(\theta^*, u^*)}$$

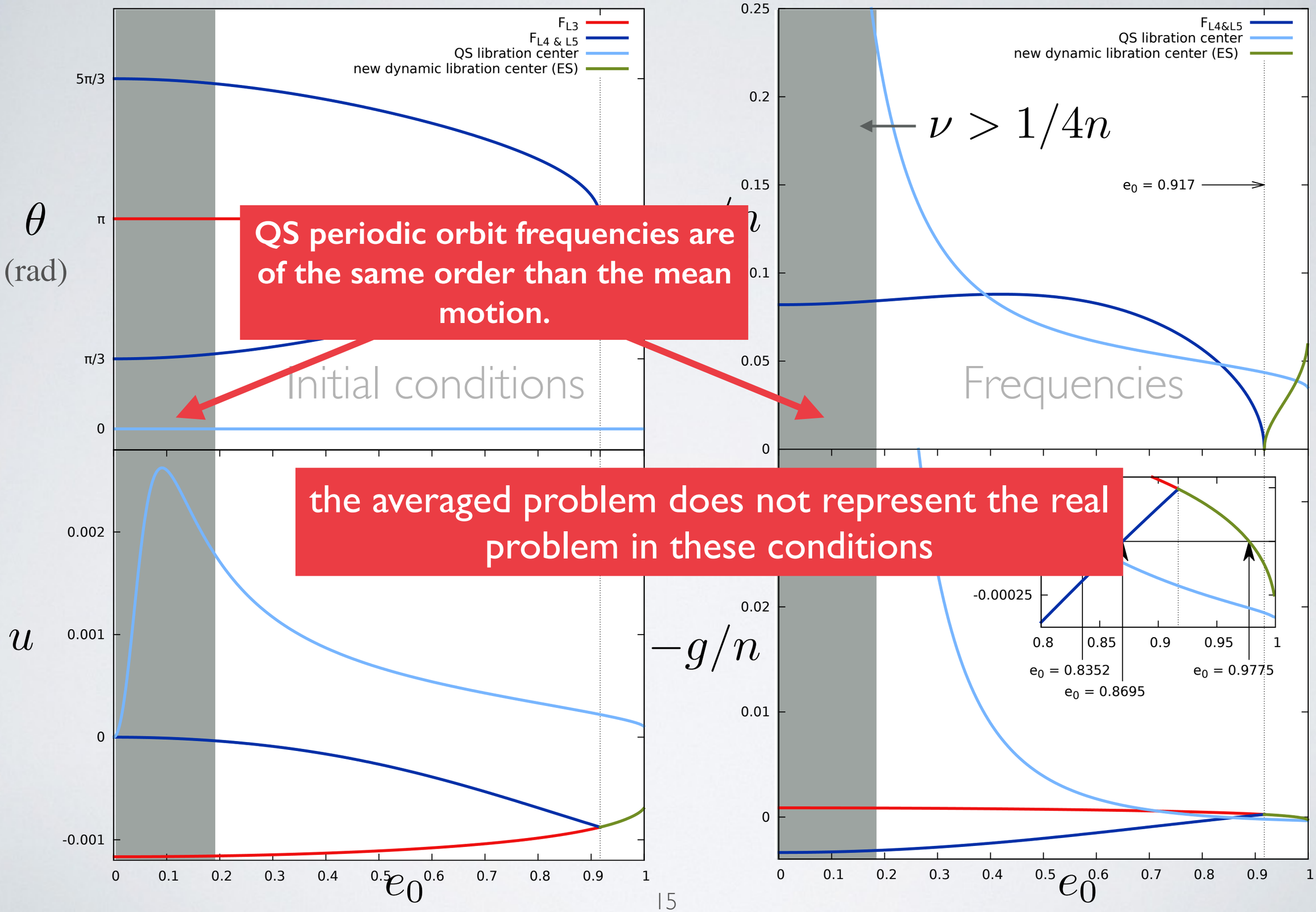
$$\frac{\partial \overline{\mathcal{H}}}{\partial \Gamma}(\theta^*, u^*; e_0) \longrightarrow \boxed{g(\theta^*, u^*; e_0)}$$

precession frequency

$$\left(\frac{\partial^2 \overline{\mathcal{H}}}{\partial x \partial y}(\theta^*, u^*; e_0)\right)_{x,\, y \in \{\theta, u\}} \longrightarrow \boxed{\nu(\theta^*, u^*; e_0)}$$

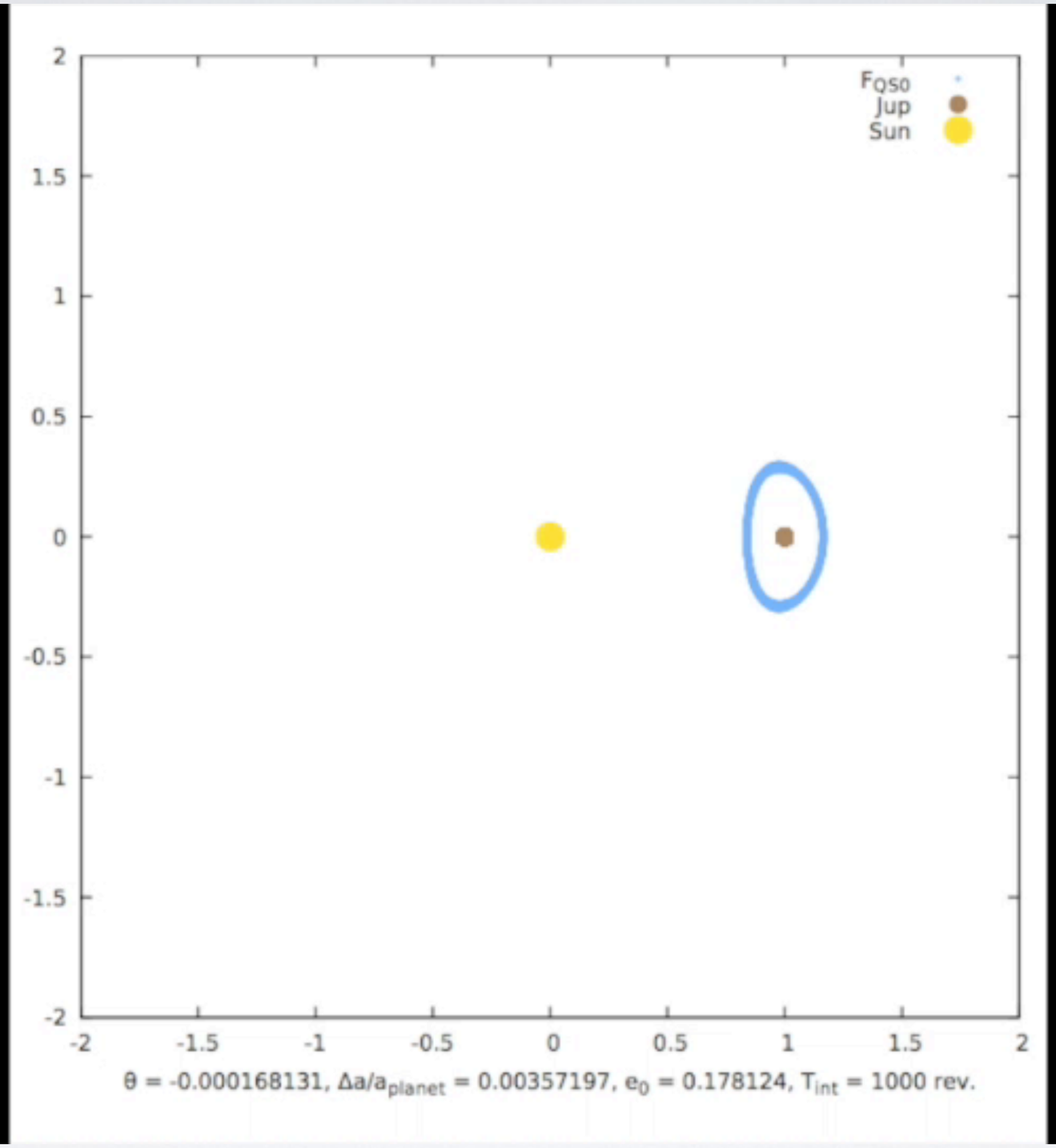
libration frequency

$e_{pl} = 0,\ \epsilon = 1/1000$

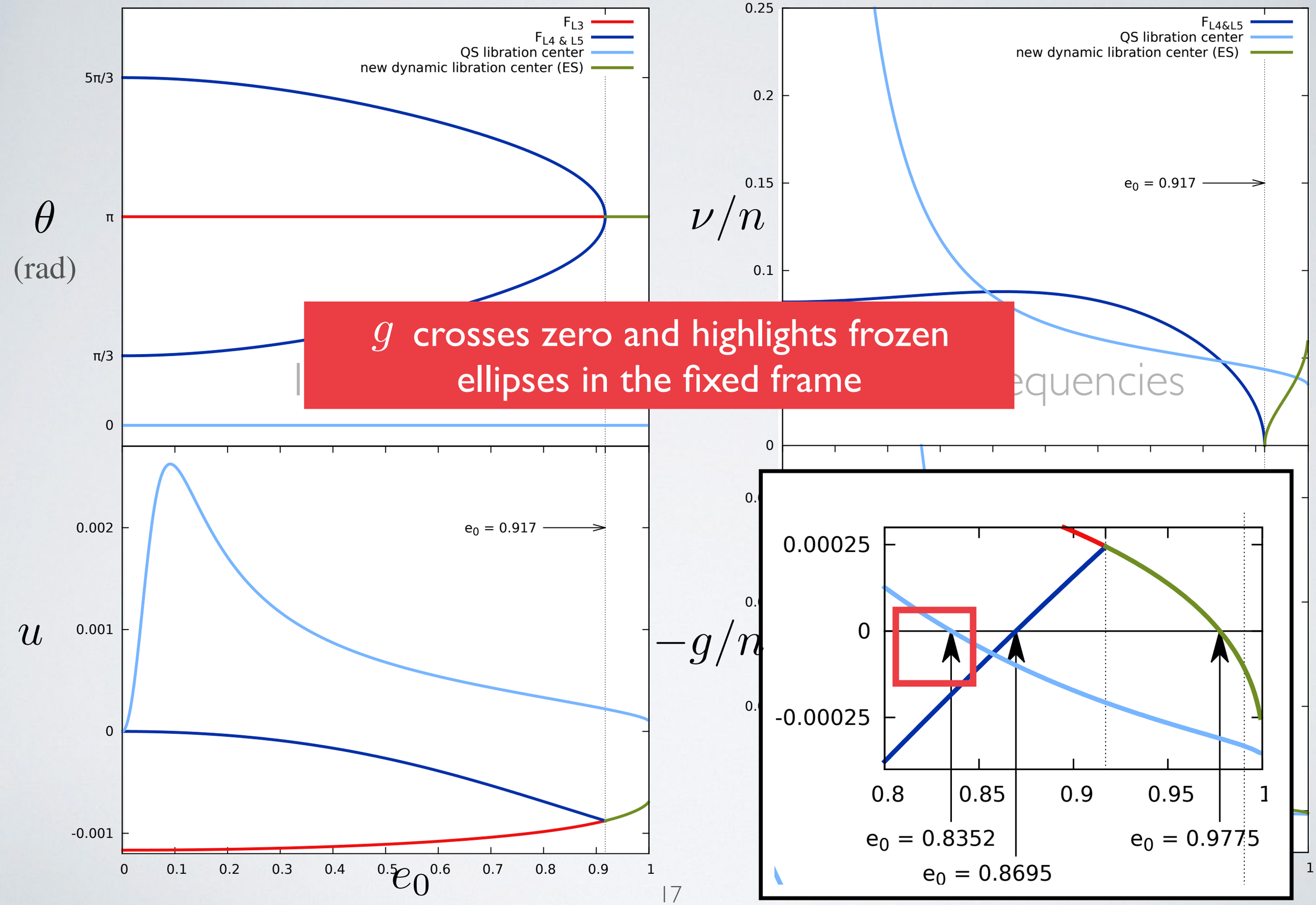


QS periodic orbit frequencies are of the same order than the mean motion.

the averaged problem does not represent the real problem in these conditions


F_{QS0}
Jup
Sun
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
θ = -0.000168131, $\Delta a/a_{planet}$ = 0.00357197, e_0 = 0.178124, T_{int} = 1000 rev.

$e_{pl} = 0,\ \epsilon = 1/1000$

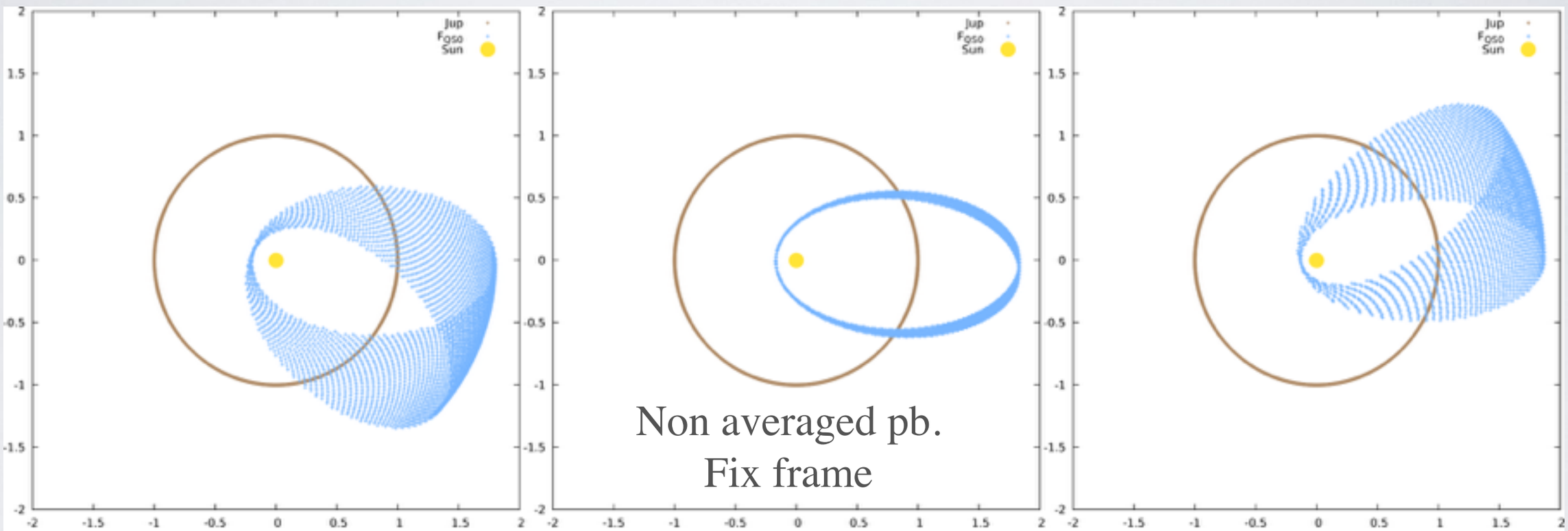


g crosses zero and highlights frozen ellipses in the fixed frame

$e_{pl} = 0,\ \epsilon = 1/1000$

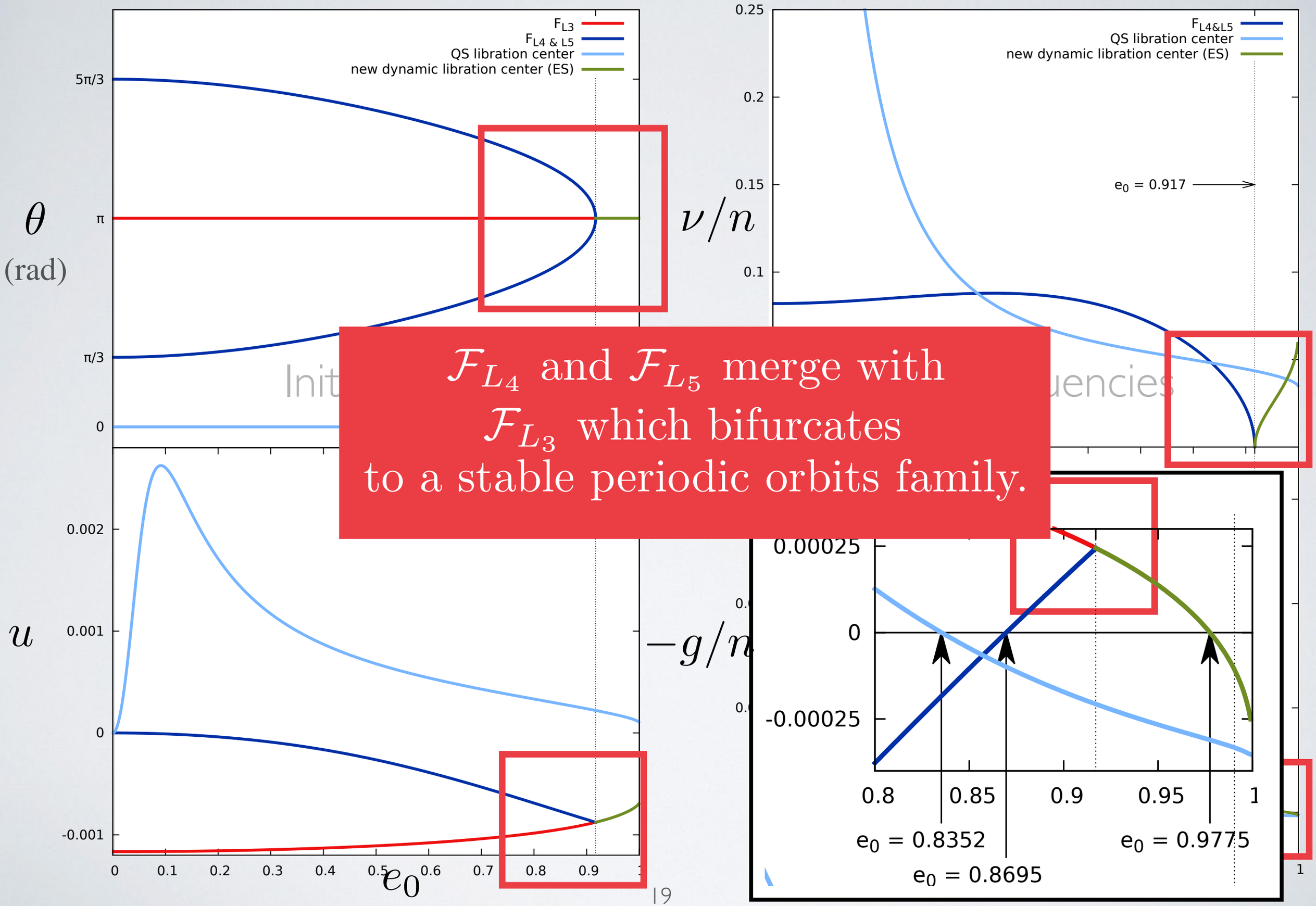
QS frozen ellipse

$\{e_0 = 0.80\}$ $\{e_0 = 0.83\}$ $\{e_0 = 0.87\}$

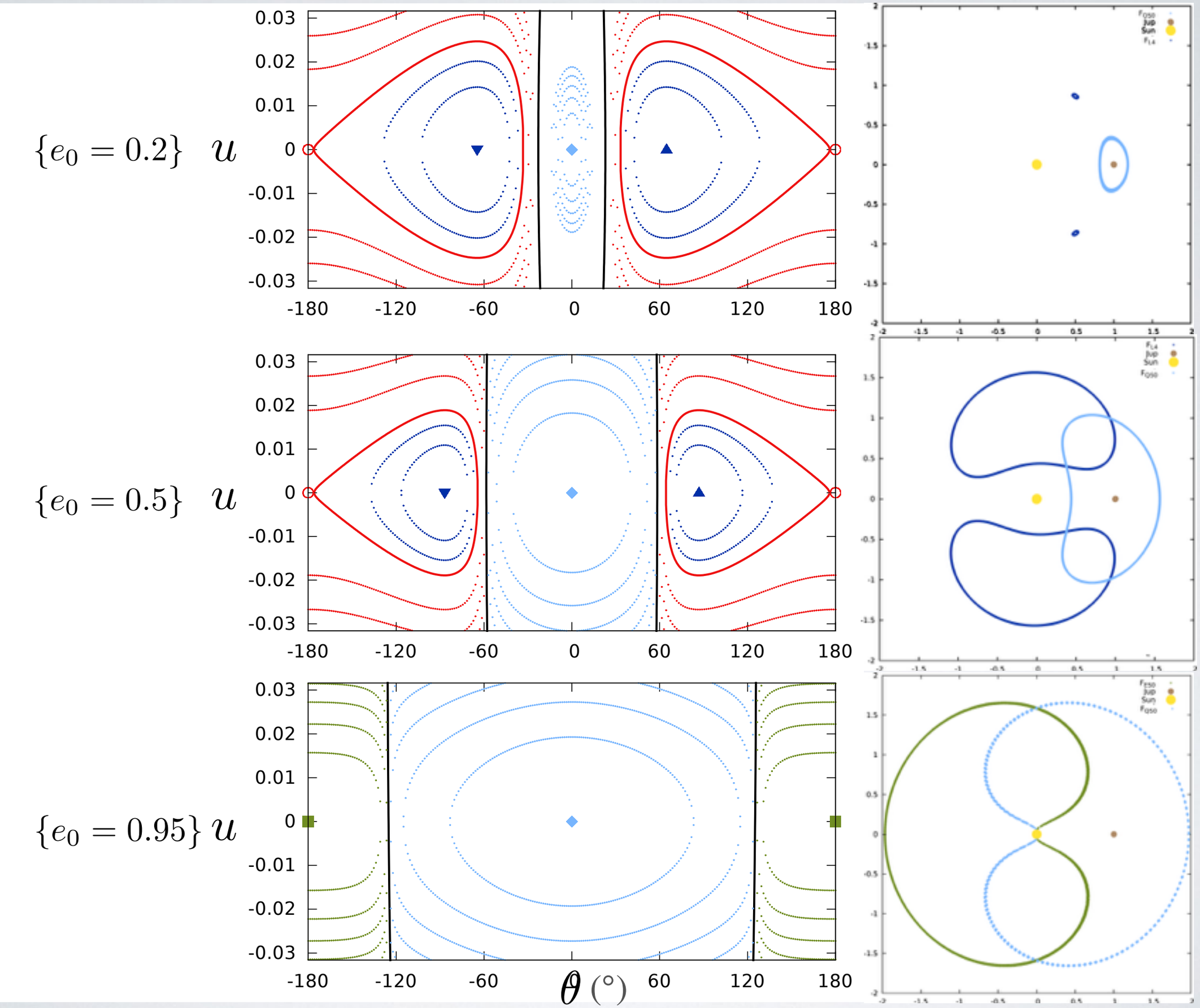


$T_{int} = 1000 \times 2\pi$

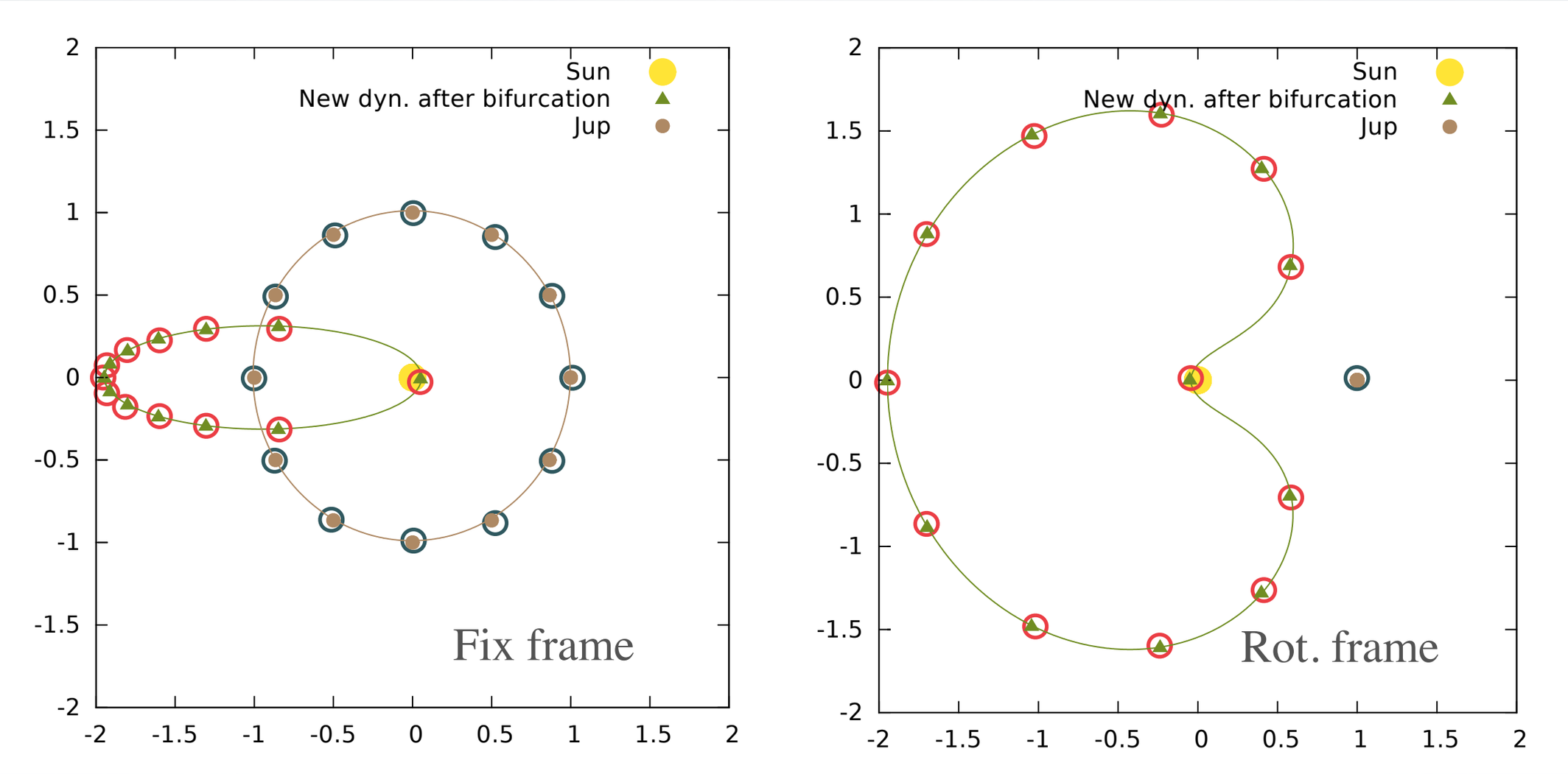
$e_{pl} = 0,\ \epsilon = 1/1000$



$e_{pl} = 0,\ \epsilon = 1/1000$ Averaged problem: (θ, u) proj. Non avgd pb. Rot. frame



New dynamic after bifurcation



in heliocentric orbital elements:

- *co-orbital motion with the secondary libration around* $\theta = \lambda - \lambda_{pl} = \pi$
- *seem satellized around L3 point in rot. frame*

CONCLUSIONS

QS Motion

- low values of eccentricity: the averaged problem does not represent the real problem (frequencies ~ n).

- stability in the circular case (domain between collision curve).

- QS frozen ellipse close to $e_0 = 0.8352$ ($g = 0$).

TP & HS Motion

- unexpected result: FL3, FL4 and FL5 merge ($e_0 = 0.918$).

- bifurcation to a stable periodic orbits family.

- appearance of a new type of orbits: libration around $\theta = \pi$